

STOCHASTIC MODELING OF TILLAGE-INDUCED AGGREGATE BREAKAGE

L. E. Wagner, D. Ding
MEMBER STUDENT MEMBER
ASAE ASAE

ABSTRACT. A Markov chain-based, two-parameter model was developed to predict tillage-induced influences on soil aggregates. Model parameters for several tillage operations were identified on two soil types with a downhill, simplex, multidimensional, minimization approach. Simulation results suggest that the crushing model can predict tillage-induced aggregate crushing and that average prediction errors are within 3% for the limited cases of verification. This study indicates that the stochastic simulation is better than the conventional deterministic method in estimating the tillage effects on soil aggregate size distribution due to apparent randomness of variability in the field data.

Keywords. Tillage, Stochastic models, Aggregates.

The Wind Erosion Prediction System (WEPS), presently being developed by the Agricultural Research Service, USDA (Hagen, 1991), requires a submodel component for simulating the effects of various tillage and management operations performed on agricultural soils. As pointed out by Cole (1988), one of the major tillage actions on soil is crushing or breaking of soil aggregates. The objective of this study was to model the crushing effect on aggregates under different soil conditions and tillage practices. More specifically, the task was to develop a simulation model based on field collected pre-tillage and post-tillage samples of aggregate size distribution. The model will then be incorporated into the WEPS submodel component, where post-tillage aggregate size distribution values will be predicted from pre-tillage aggregate size distribution, tillage operation being performed, soil type, and possibly other factors.

A deterministic model (i.e., all the components of the model are deterministic), equation 1, originally developed for modeling solid grinding processes such as coal and rocks (Austin, 1971/1972), was first attempted to model the soil aggregate crushing process. Equation 1 is derived based on the conservation of mass principle, i.e., the mass of material in size class i after one stage of breakage equals the sum of material broken into size class i from larger size classes plus the original material in size class i minus the material broken out of size class i .

$$w_1[i] = (1 - s_i) w_0[i] + \sum_{k=i+1}^N (B_{k,i+1} - B_{k,i}) s_k w_0[k] \quad (1)$$

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The authors are Larry E. Wagner, Agricultural Engineer, USDA-Agricultural Research Service, Wind Erosion Research Unit, Manhattan, KS; and Dajiang Ding, Graduate Research Assistant, Agricultural Engineering Dept., Kansas State University, Manhattan.

where

$w_0[i]$ = the initial mass aggregate size fraction distribution

$w_1[i]$ = the final mass aggregate size fraction distribution

$B_{k,i}$ = the cumulative distribution function (mass fraction of material, broken from size class k that falls into size classes smaller than size interval i)

s_i = the selected fraction of size interval i for breakage

s_k = the selected fraction of size interval k for breakage

N = the total number of sieve sizes

Many functional forms for $B_{k,i}$ and s_i (Austin, 1984) were tested with equation 1 and the back-calculation procedures used by Gupta et al. (1981). The best modeling results were achieved when $B_{k,i}$ and s_i assumed the following form:

$$B_{k,i} = \left(\frac{gmd_i}{gmd_k} \right)^\alpha \quad s_i = s_{\max} \left(\frac{gmd_i}{gmd_{\max}} \right)^\beta \quad (2)$$

where

gmd_i, gmd_k = the geometric mean diameter of size class i and k , respectively, with $gmd_i < gmd_k$

s_{\max} = the percent of breakage from the largest size class

α, β = parameters

However, it was determined that the equation 1 based deterministic crushing model was inadequate due to the following observations: 1) it was very difficult to get a consistent estimation of α and β because of significant variance in the field data; 2) it was hard to choose appropriate functional forms for $B_{k,i}$ and s_i since information on their individual terms were not obtainable from the field data and the quantity of data limited the number of parameters that could potentially be used; and

Table 1. Selected soil properties

Property	Eudora Silt Loam	Kimo Silty Clay Loam
Textural composition		
Sand (2.0-0.05 mm) (%)	29.1	20.0
Silt (0.05-0.002 mm) (%)	54.5	44.0
Clay (< 0.002 mm) (%)	16.4	36.0
Water content at		
-33 J / kg (g / g)	0.165	0.249
-1000 J / kg (g / g)	0.061	0.140
Standard Proctor test		
Maximum density (Mg / m ³)	1.58	1.53
Optimum water content (g / g)	0.155	0.192
Organic matter (%)	1.50	2.20
pH	6.30	6.50
Exchangeable cations (ppm)		
K	149	350
Ca	1698	3470
Mg	208	330
Na	8	14
Al	0	0

3) parameters α and β were strongly dependent on the value of s_{\max} , which changes significantly with soil conditions.

As a result of the difficulties with the deterministic model, a stochastic approach was pursued to model the crushing process because: 1) the field data contained significant variance which could be treated as a random process, and 2) a unified treatment would enable us to avoid the complications encountered with a separate $B_{k,i}$ and s_i . This article describes the effort in that stochastic model development.

EXPERIMENTAL DATA SETS

The aggregate size distribution (ASD) data sets used in this study were obtained from several experimental field studies, some of which have been published (Tangie et al., 1990; Ambe, 1991; Wagner et al., 1992). All experiments were conducted on two soils (table 1): Kimo silty clay loam (clayey over loamy, montmorillonitic, mesic Aquic Hapludolls) and Eudora silt loam (coarse-silty, mixed, mesic Fluventic Hapludolls) at the Kansas River Valley Experiment Field near Topeka, Kansas. Individual objectives and experimental designs of these field studies varied; however, each of these experiments contained pre- and post-tillage ASD measurements of which some were suitable for use in the development of the stochastic aggregate crushing model. Of the suitable data sets, half of the measurements were used for determination of model parameters and the remaining measurements for determining accuracy of model predictions.

The quantity and number of replications of ASD samples used varied among the experiments, but all were collected and processed in the same manner. Pre-tillage ASD samples (approx. 10 kg) were collected from the first 15 cm and post-tillage ASD samples were obtained from within the resulting tillage tool processing depth. These samples were extracted at randomly selected locations (between wheel tracks) in each plot using a 30- × 23-cm flat, square-cornered shovel, as described by Chepil (1962), and placed in 46- × 30- × 6-cm plastic tubs. All aggregate

size distribution samples were air-dried in a greenhouse prior to sieving with a modified combined rotary sieve (Lyles et al., 1970).

Suitable ASD data sets were available for a variety of tillage implements, although the size of the data sets varied among them, from having multiple data sets for both soils to single data sets for only one soil. All speeds and depths were typical for each respective tillage operation. The ASAE Standard S414 tillage implement names, descriptions, speeds, and processing depths were:

- Offset disk harrow — 45 cm diameter blades with a 30 cm inter-disk spacing (8 km/h, 16 cm).
- Chisel plow — two ranks of 36 cm deep, ridgedly mounted curved shanks with point chisels, having an inter-tool rank spacing of 60 cm (8 km/h, 19 cm).
- Field cultivator — three ranks of spring teeth with an inter-tool rank spacing of 45 cm (9 km/h, 10 cm).
- Rotary tiller — garden tractor powered rotary tiller with a blade radius of 16 cm (4 km/h, 18 cm).

MODEL DESCRIPTION AND IDENTIFICATION

The stochastic model* for the crushing process was a Markov† chain model (Bhat, 1984), which can be stated as follows in the context of the soil aggregate crushing process:

A soil aggregate is assumed to consist of many particles, with each having an infinitesimal volume and a unit mass. The soil particles can travel only downward from a larger aggregate size class to smaller aggregate size classes after each tillage pass (crushing of an aggregate). If a size class is called a "state", then the transition of soil particles from one state to another can be treated as a completely random event. A probability matrix, $P[i,j]$, can be constructed for all possible transitions occurring in the soil when its aggregate size distribution (mass fractions across different size classes) shifts or transfers from $w_0[i]$ to $w_1[k]$ (0 to $i-1$) after one crushing stage (tillage pass). $P[i,j]$, often called a *transition matrix*, maintains the properties of a Markov chain and does not change with the number of tillage passes performed but depends on the type of tillage and the specific soil conditions.

Mathematically, the Markov chain-based crushing model is of the form:

$$w_1[i]_{(1 \times n)} = w_0[i]_{(1 \times n)} P[i,j]_{(n \times n)} \quad (3)$$

The effectiveness of the model relies on how accurately the transition matrix, $P[i,j]$, can be estimated. According to the model statement, the transition matrix can be generalized as a lower triangular matrix, where the states with smaller index values correspond to the smaller aggregate size classes (size intervals) and vice versa.

* A stochastic model has at least one component that will be treated as exhibiting random behavior.

† A Markov process is one in which the next "state" is dependent only on the present "state" and is independent of any previous "state".

$$P[i,j] = \begin{pmatrix} p_{11} & 0 & \dots & 0 \\ p_{21} & p_{22} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{i1} & \dots & p_{ij} & \dots & p_{ii} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \dots & p_{nn-1} & p_{nn} \end{pmatrix}_{n \times n} \quad (4)$$

Because it is almost impossible to estimate each transition probability (p_{ij}) individually, we assumed that the p_{ij} follows a binomial distribution[‡] as shown in equation 5. The binomial distribution is a typical discrete probability distribution function.

$$p_{ij} = \binom{i-1}{j-1} p_i^{j-1} (1-p_i)^{i-j} \quad (5)$$

$i = 1, 2, \dots, n; j = 1, 2, \dots, i$

In equation 5, p_i is defined as the probability function of breakage, which has a value within the interval [0,1] and generally can be expressed as an algebraic function of sieve size (x_i) and a number of parameters, $\alpha_1, \alpha_2, \dots, \alpha_m$.

$$p_i = f(x_i, \alpha_1, \alpha_2, \dots, \alpha_m) \quad (6)$$

The probability function of breakage (p_i) reflects how much breaking is occurring in the aggregate size class i . A large p_i indicates a small percentage of soil aggregates of size class i that will break into smaller size classes. If $p_i = 1$, then no aggregates of size class i are being broken down, and if $p_i = 0$, then all of the aggregates in size class i are being broken down into smaller size classes.

It is presumed that p_i is related to the tillage tool, speed and depth, soil properties and conditions, and sieve cut sizes used in measuring $w_0[i]$ and $w_1[i]$. Therefore, the α_i parameters in equation 6 were expected to be functions of those conditions.

$$p_i = f(x_i, \alpha, \beta) \quad (7)$$

In this analysis, the focus was on a two parameter representation of a breakage probability function such as equation 7, primarily because: 1) identifying multiple parameters was complicated; and 2) the size of the data file for model parameterization was small (eight pairs of data points as shown in table 2 for an eight-cut rotary sieve).

[‡] This probability distribution of Bernoulli trials consists of repeated independent trials. Each trial has two possible outcomes, and the corresponding probability remains the same for all trials.

Table 2. A sample crushing data form

Soil type	Silt loam								
Tillage tool	Offset disk								
Experiment index	91-8-ASD111.1								
Sieve-size index (i)	0	1	2	3	4	5	6	7	8
Sieve-size x_i (mm)	0.01	0.42	0.84	2.0	6.36	19.05	44.45	76.2	152.4
ASD on mass basis									
Before tillage ($w_0[i]$) (%)	7.8	2.9	6.5	14.9	24.4	25.8	10.0	7.6	
After tillage ($w_1[i]$) (%)	7.9	4.5	10.3	20.4	26.3	20.8	6.9	2.9	

Therefore, two parameters were chosen to reflect data variability.

The model identification included finding a suitable p_i function and then searching for function parameters for different tillage tools and soil conditions. Several functional representations of p_i were identified. Initial study suggested that four functions were most promising:

$$p_i = \alpha \left(1.0 - \beta \frac{x_i}{x_{\max}} \right) \quad (8)$$

$$p_i = \alpha \left(\frac{x_i}{x_{\max}} \right)^\beta \quad (9)$$

$$p_i = 1.0 - \exp \left(-\alpha + \beta \frac{x_i}{x_{\max}} \right) \quad (10)$$

$$p_i = \frac{1.0}{1.0 + \exp \left(-\alpha + \beta \frac{x_i}{x_{\max}} \right)} \quad (11)$$

where

- x_i = individual sieve sizes
- x_{\max} = the maximum sieve size
- α, β = the parameters to be determined

The back-calculation procedure used to estimate α and β in the above equations was based on known $w_0[i]$ and $w_1[i]$. It is a multi-dimensional minimization of the target function:

$$\varepsilon = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(w_1[i] - \sum_{j=1}^n w_0[j] [p_{ji}] \right)^2} \quad (12)$$

Two types of multi-dimensional minimization algorithms were used in the parameter identification. One was the gradient-based method which required calculation of derivatives for the function (Press et al., 1988). We found that this method converged slowly and was very sensitive to the initial conditions. The other type minimization algorithm used included gradient-free methods, such as 1) the Powell minimization, and 2) the downhill simplex method. We found that the two gradient-free methods gave almost identical results and showed less

Table 3. Parameters of crushing model for four types of tillage tools

Tillage Implement		Silt Loam			Silty Clay Loam		
		α	β	No. Data Sets	α	β	No. Data Sets
Rotary Tiller	Mean	1.40	-1.20	5	1.50	0.56	3
	S.D.	0.60	1.70		0.30	0.55	
Disk Harrow	Mean	2.80	0.75	9	4.30	2.00	8
	S.D.	0.50	0.28		1.60	1.50	
Chisel Plow	Mean	—	—	—	2.40	-2.00	4
	S.D.	—	—	—	1.20	4.60	
Field Cultivator	Mean	3.00	-0.22	1	3.00	1.80	2
	S.D.	—	—		0.90	0.50	

Table 4. Rules of thumb for determining degree of crushing based on model parameters

Rule 1	If α is small	$\alpha < 2$	Then a high percentage of aggregates from all size classes is being crushed or broken down.
Rule 2	If α is not small and β is approximately one-half of α	$\alpha \geq 2$ and $\beta \approx 1/2 \alpha$	Then a high percentage of large aggregates is being crushed or broken down.
Rule 3	If α is large and β is small	$\alpha \geq 3$ and $\beta \leq 1.5$	Then only a small amount of aggregates is being crushed.

sensitivity to the initial conditions. Most of the calculations were carried out with the downhill simplex method. A computer program was written in the C language and based on code published by Press et al. (1988).

Data used for model parameterization were extracted from experimental field data. The data were grouped into the format shown in table 2 after computing the mass percentage distributions, removing data sets containing apparent errors and averaging multiple observations for each measurement. Because of field data collection problems and randomness associated with the tillage operation and field conditions, relatively large variances still occurred in the data sets.

RESULTS AND DISCUSSION

The most suitable functional representation for p_i was found to be the form of equation 11 as shown in equation 13.

$$p_i = \frac{1.0}{1.0 + \exp \left(-\alpha + \beta \frac{\text{gmd}_i}{\text{gmd}_{\max}} \right)} \quad (13)$$

where $i = 1, 2, 3, \dots, n$ (number sieves)

§ For a rotary sieve of n sieves, x_0 and x_{n+1} are arbitrary; minimum and maximum aggregate sizes are assumed to exist in the data. The values used in this analysis were 0.01 mm and 152.4 mm, respectively, as shown in table 2.

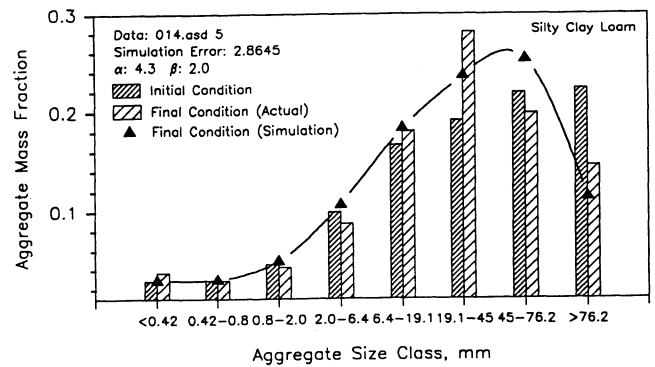


Figure 1—Crushing by offset disk on silty clay loam with many large aggregates.

gmd_i = geometric means of x_{i-1} and x_i (geometric mean diameter of aggregates in each size class)

gmd_{\max} = geometric mean of x_n and x_{n+1} (geometric mean diameter of aggregates in largest size class)

Parameter α reflects the crushing of all soil aggregates regardless of size. As α decreases, the percentage of soil aggregates breaking down increases. Parameter β reflects the unevenness of crushing among aggregates in different size classes. Large β values mean that crushing mainly affects the large soil aggregates.

The parameters in the model represented by equations 3, 4, 5, and 13 were estimated for the four tillage tools using the back calculation procedure. The results are listed in table 3. Although the parameters in table 3 are derived from a limited number of field data sets, they gave good indications of how much crushing each tillage tool caused. Based on the α values for the silt loam, the rotary tiller produced the most overall crushing and the field cultivator produced the least. The disk harrow had more crushing of larger aggregates occurring and is reflected by its relatively large β value. For the silty clay loam, the rotary tiller still generated the most crushing; whereas, the disk harrow and field cultivator exhibited strong effects on large aggregates (large β values).

To judge how much crushing is caused by a tillage implement based upon its two parameters, the rules of thumb in table 4 can be applied.

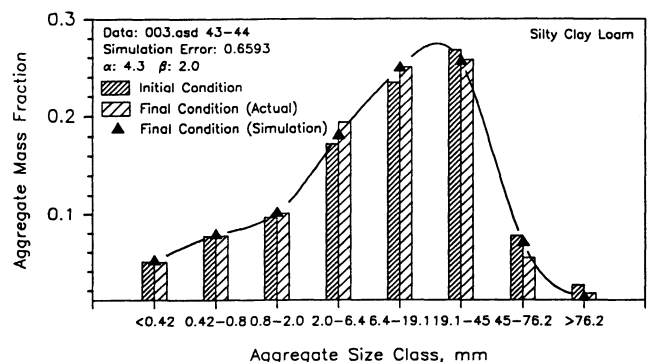


Figure 2—Crushing by offset disk on silty clay loam with few large aggregates.

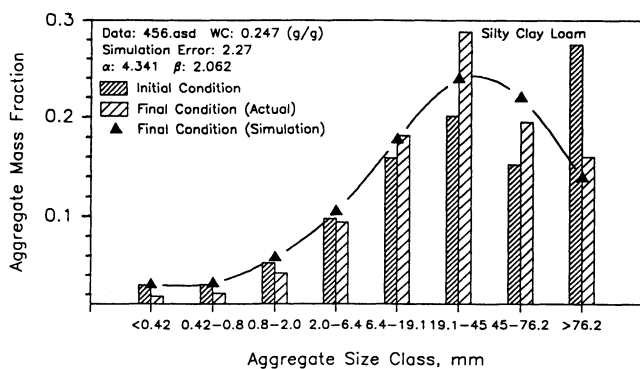


Figure 3—Crushing by offset disk on silty clay loam at “high” water content.

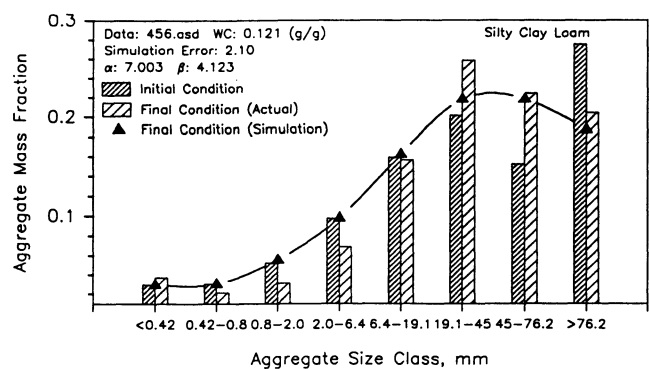


Figure 5—Crushing by offset disk on silty clay loam at “low” water content.

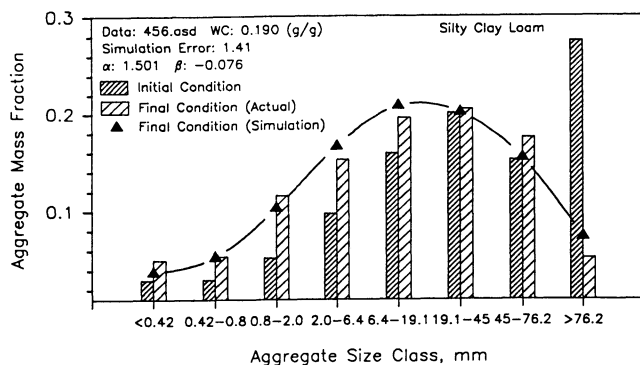


Figure 4—Crushing by offset disk on silty clay loam at “optimum” water content.

The relatively large standard deviations shown in table 3 that were encountered in the parameter identification processes were caused by the variability in the field data. Major problems can exist in collecting valid field data. For example, (a) measuring the ASD of compacted soil, (b) obtaining accurate estimates of field ASD values of sandy soils because of aggregate susceptibility to damage during the rotary sieving process, and (c) drying effects on ASD samples obtained at different water contents.

With the parameters identified, the crushing model was further verified with limited data sets from other field experiments conducted on the same sites. Figures 1 and 2 show simulation results for the offset disk on a silty clay site. Results show that the offset disk causes significant breakup of large clods when they are present (fig. 1) and causes little breakup of the same soil when large clods are not present (fig. 2).

The model reflects the influence of soil water contents at tillage as demonstrated by Wagner et al. (1992). Using their data, the parameters of the model were identified for the two soils under “high”, “optimum”, and “low” water contents. The results for the silty clay loam are shown in figures 3 through 5, where it can be seen that the most breakup of soil aggregates occurred at the optimum water

content (fig. 4) and the α parameter of the model is significantly lower than those at the high and low water contents. Since the effect of water content on the silt loam is not as strong as on the silty clay loam, the change in α under the three water contents was not significant (figs. 6 through 8). However, α was again lowest at the optimum water content (fig. 7). This suggests that the parameters of the crushing model could be expanded to be a function of soil water content and potentially reduce the variability observed in determining these parameter values, especially in higher clay content soils.

The crushing of the silt loam soil by the offset disk displayed different characteristics than the crushing of the silty clay loam as shown in figures 3 through 8, because the silt loam soil contained fewer large aggregates. The figures also indicate that the model can predict the disk-induced crushing processes reasonably well. Prediction errors, defined as the average error across all the size classes, are within three percent for all cases. However, the crushing model needs to be further verified with data from other tillage experiments.

The stochastic crushing model also can be used in reverse to estimate pre-tillage aggregate mass fraction distributions based on the measurement of the post-tillage distribution by reversing the transition matrix, $P[ij]$. Another possible application is modeling a series of crushing events as a single operation by multiplying together the transition matrix associated with different tillage devices.

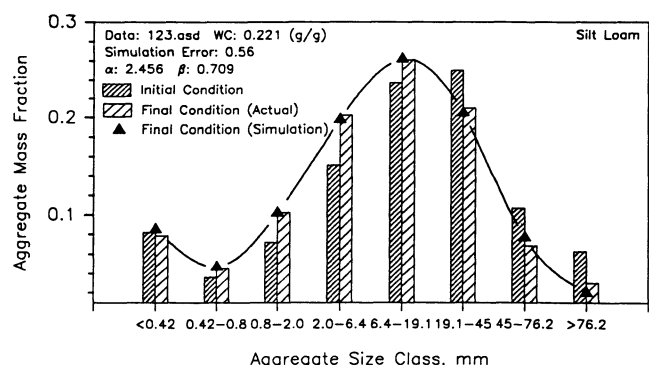


Figure 6—Crushing by offset disk on silt loam at “high” water content.

II “Optimum” water content refers to the water content that will produce the maximum soil bulk density from a standard Proctor test. “Low” water content is a value 40 to 60% lower than the “optimum” and the “high” water content refers to a value 40 to 60% greater than the “optimum”.

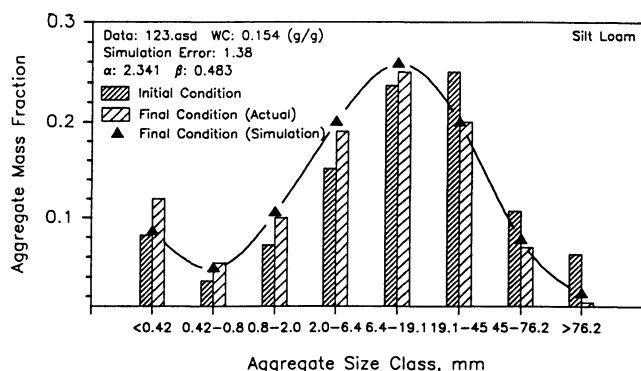


Figure 7—Crushing by offset disk on silt loam at “optimum” water content.

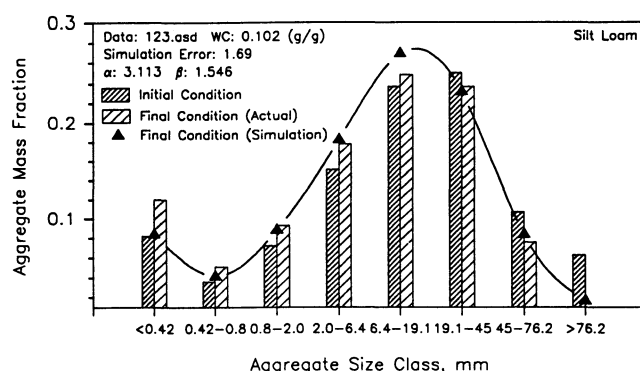


Figure 8—Crushing by offset disk on silt loam at “low” water content.

SUMMARY AND CONCLUSIONS

The equation 1 based deterministic crushing model failed to adequately describe the tillage-induced aggregate crushing because of the difficulties in selecting appropriate functions and identifying their associated parameters. The stochastic approach was taken because the crushing process could be described as a Markov process, allowing a relatively simple probability function to be selected. This approach was successful in that it gave consistent estimation of the parameters involved. However, to increase the precision of parameter identification, field data measurement and collection procedures will require improvement to reduce the variability in pre- and post-tillage aggregate size distribution data. Also, a major challenge exists to parameterize the model for various soil and tillage conditions. This will require a large amount of field data from well designed and executed experiments.

The study can be summarized as follows:

- Tillage-induced soil aggregate crushing can be approximated as a random process. The Markov chain-based, two-parameter, stochastic crushing model gave consistent and fairly accurate estimations of this random process.
- The physical meaning of the parameters were explored. Empirical rules explaining how these parameters characterize the crushing process were derived.
- Simulation of disk-induced crushing, based on the limited data analyzed, contained simulation errors within 3%.
- The stochastic crushing model was successful at simulating tillage-induced aggregate crushing on two types of soils (under an aggregated condition) and gave a consistent estimation of the parameters involved.
- More tillage data, with other implements under various soils and soil conditions, are needed to extend the application of this modeling approach.

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